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OBLIQUE HYPERVELOCITY IMPACT SIMULATION FOR MULTI-LAYER ORBITAL DEBRIS SHIELDING

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ABSTRACT

Particle-based hydrodynamics models offer distinct advantages over Eulerian and Lagrangian hydrocodes in particular shock physics applications. Particle models are designed to avoid the mesh distortion and state variable diffusion problems which can hinder the effective use of Lagrangian and Eulerian codes respectively. However existing particle-in-cell and smooth particle hydrodynamics methods employ particles which are actually moving interpolation points. The latter distinction has been emphasized in the more recent development of element-free Galerkin theory. As a result general formulations of all of the aforementioned methods are based on the partial differential equation forms of the continuum balance laws which underlie conventional Eulerian and Lagrangian schemes. An alternative modeling methodology, based on the application of Hamilton's equations to a system of deforming physical particles, provides a fully Lagrangian, energy-based approach to shock physics simulations. Neither interpolations of field variables nor continuum balance laws are used to establish the state equations for the particle system. Mechanical and thermal interaction of the particles is accounted for by nonholonomic constraints which determine both particle entropy evolution and particle collision loads. Application of the method is illustrated by simulation of wall shock, Whipple shield, and multi-plate shield impact problems. A three dimensional, vectorized and autotasked implementation of the particle model presented here has been coded for application to orbital debris shield design simulations.

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INTRODUCTION

Most numerical models for shock physics simulations (Anderson, 1987) have been based on continuum balance laws for mass, momentum, and energy formulated in Eulerian, Lagrangian, or arbitrary Lagrangian-Eulerian (ALE) reference frames (Benson, 1992). Examples are the Eulerian finite volume code CTH (McGlaun et. al, 1990), the Lagrangian finite element code DYNA3D (Goudreau and Hallquist, 1982), and the arbitrary Lagrangian-Eulerian code RHALE (Budge and Peery, 1993). Such codes have seen extensive and successful application to a wide range of problems. Despite this success, the effective use of any continuum based model in some astrophysics (Benz et. al, 1986) and engineering design (Hertel, 1993) applications has proven quite difficult. For example, mesh distortion in Lagrangian finite element codes and difficulties in tracking highly fragmented solid bodies in Eulerian hydrocodes have made continuum-based simulations of particular hypervelocity impact problems impractical (Fahrenthold, 1993 and 1995). The limitations of continuum based codes in certain shock physics applications has been in part responsible for the development of alternative particle-based methods for hydrodynamics simulations.

Most particle-based hydrodynamics models employ either a particle-in-cell (PIC) or a smooth particle hydrodynamics (SPH) approach. The basic PIC method (Monaghan, 1985) and its extensions (Sulsky et. al, 1994) employ nondeforming mass fixed particles and a space fixed grid. Mapping of field variables back and forth between the particles and the grid serves to define the gradients of field variables, establish the update of the particle properties, and advect the particles with their associated internal states. The basic SPH method (Monaghan, 1992) and its extensions (Libersky et. al, 1993) again employ nondeforming mass fixed particles, in this case with moving interpolation functions. In SPH the moving interpolation functions define the gradients of field variables, allow for updating of the particle properties, and determine the motion of the particles with their associated internal states. As emphasized in recent work on the development of element-

free Galerkin (EFG) methods (Lu et. al, 1995), the 'particles' of the PIC, SPH, and EFG formulations are in fact moving interpolation points. As a result, general formulations of all the above methods rely at least in part on the partial differential equations of continuum mechanics in establishing a system level model - usually the mass, momentum, and energy balance laws which underlie conventional Eulerian finite difference and Lagrangian finite element models of shock physics problems.

As an alternative to existing particle based methods, the present report applies Hamilton's equations to a system of translating, deforming, and thermomechanically interacting physical particles, to obtain a three dimensional hydrodynamic model for shock physics simulations. The method is energy based and hence simple to formulate, with no need to consider continuum balance laws, interpolation functions, or weighted residual solution techniques. Since the particle kinematics, energy functions, and constraints are fully Lagrangian in form, the method avoids the diffusion problems typically associated with interpolation on space fixed meshes (McGlaun et. al, 1990) and the stability problems observed with some moving interpolations (Swegle et. al, 1995). Although previous work has made limited use of entropy states (Monaghan, 1992 and Brown, 1981), total entropy variables appear here as generalized coordinates for the particles, subject to nonholonomic thermal constraints describing heat conduction and energy dissipation. Additional nonholonomic constraints on the particle coordinates and deformation gradients are introduced, in order to represent mechanical interaction of the particles. The latter approach avoids the introduction of essentially Eulerian potential energy functions (Monaghan, 1988) or the remapping of data between the particles and a grid (Sulsky et. al, 1994) in order to quantify collision forces. Finally the fully Lagrangian frame of reference used here means that no mixing or partial pressures expression (McGlaun et. al, 1990 and Monaghan and Kocharyan, 1995) is needed to represent the multi-material case.

The succeeding sections are organized as follows. First the particle kinematics are specified. Next the system Hamiltonian is formed by defining kinetic and potential energy

expressions. This identifies the generalized coordinates and momenta which serve as state variables. Next the viscosity and heat conduction models are formulated, as well as the thermal and mechanical constraints which act on the system. Finally the preceding results are applied to establish Hamilton's equations, specifically a first order state space model for the particle system, in explicit rate form. Simulation of wall shock, Whipple shield impact, and multi-plate shield impact problems illustrate numerical application of the method.

KINEMATICS

The physical system modeled here is a collection of 'n' deforming spherical particles, interacting in both the mechanical and thermal domains, with the fixed mass of particle 'i' denoted by $m^{(i)}$ (i = 1, 2, ..., n). For homogeneously deformed particles, the position ($\mathbf{x}$) and velocity ($\dot{\mathbf{x}}$) of a material point in particle 'i' are expressed as

$$\mathbf{x} = F^{(i)} (\mathbf{r} - \mathbf{r}^{cm(i)}) + \mathbf{c}^{(i)} \quad ; \quad \dot{\mathbf{x}} = \dot{F}^{(i)} (\mathbf{r} - \mathbf{r}^{cm(i)}) + \dot{\mathbf{c}}^{(i)} \qquad (1a,b)$$

where $\mathbf{r}$ and $\mathbf{r}^{cm(i)}$ are the positions of the material point and the particle center of mass respectively in the reference configuration, $\mathbf{c}^{(i)}$ is the current position of the particle center of mass, $F^{(i)}$ is the particle deformation gradient, and a superposed dot denotes the total time derivative. The particle volume in the current ($V^{(i)}$) and reference ($V_o^{(i)}$) configurations are related by (Malvern, 1969)

$$V^{(i)}/V_o^{(i)} = \det(\mathbf{F}^{(i)}) = F^{(i)3} \qquad (1c)$$

For spherical particles,

$$V^{(i)} = (4\pi/3)\, h^{(i)3} \qquad (1d)$$

where the particle radius ($h^{(i)}$) is

$$h^{(i)} = h_o^{(i)} F^{(i)} \quad ; \quad h_o^{(i)} = [3V_o^{(i)}/(4\pi)]^{1/3} \qquad (1e,f)$$

Note that $\mathbf{c}^{(i)}$ and $F^{(i)}$ are generalized coordinates which describe the particle motion.

KINETIC ENERGY

The system kinetic co-energy (T^*) is

$$T^* = \sum_{i=1}^{n} T^{*(i)} \quad ; \quad T^{*(i)} = (1/2) \int_{V^{(i)}} \rho \, \dot{\mathbf{x}} \cdot \dot{\mathbf{x}} \, dV \tag{2a,b}$$

Using equations (1), exact evaluation of the integral (2b) yields

$$T^* = (1/2) \sum_{i=1}^{n} [\, m^{(i)} \, \dot{\mathbf{c}}^{(i)2} + J^{(i)} \, \dot{F}^{(i)2} \,] \tag{2c}$$

where $J^{(i)}$ is a (constant) moment of inertia calculated in the reference configuration

$$J^{(i)} = (m^{(i)}/V_o^{(i)}) \int_{V_o^{(i)}} (\mathbf{r}-\mathbf{r}^{cm(i)}) \cdot (\mathbf{r}-\mathbf{r}^{cm(i)}) \, dV_o \tag{2d}$$

The kinetic co-energy expression (2c) defines the system momenta as

$$\mathbf{p}^{(i)} = \frac{\partial T^*}{\partial \dot{\mathbf{c}}^{(i)}} = m^{(i)} \, \dot{\mathbf{c}}^{(i)} \quad ; \quad H^{(i)} = \frac{\partial T^*}{\partial \dot{F}^{(i)}} = J^{(i)} \, \dot{F}^{(i)} \tag{2e,f}$$

Finally a Legendre transform allows the kinetic energy (T) to be written as a function of the center of mass momenta ($\mathbf{p}^{(i)}$) and distributed momenta ($H^{(i)}$)

$$T = \{ \sum_{i=1}^{n} [\, \mathbf{p}^{(i)} \cdot \dot{\mathbf{c}}^{(i)} + H^{(i)} \, \dot{F}^{(i)} \,] \} - T^* \tag{2g}$$

$$T = (1/2) \sum_{i=1}^{n} [\, m^{(i)-1} \, \mathbf{p}^{(i)2} + J^{(i)-1} \, H^{(i)2} \,] = T(\mathbf{p}^{(i)}, \, H^{(i)}) \tag{2h}$$

The distributed momentum accounts for the kinetic energy of expansion or compression of the particles.

INTERNAL ENERGY

The Hamiltonian approach used here incorporates general thermal and mechanical dynamics, hence the conserved potential is the total internal energy (U). For a mass fixed

system with the particle kinematics previously outlined, the exact expression for the total internal energy is

$$U = \sum_{i=1}^{n} m^{(i)}\, u^{(i)}(\rho^{(i)}, s^{(i)}) \tag{3a}$$

where $u^{(i)}$, $\rho^{(i)}$, and $s^{(i)}$ are the internal energy per unit mass, density, and entropy per unit mass for particle 'i'. Since the latter two variables are related to the total particle entropy ($S^{(i)}$) and deformation gradient by

$$S^{(i)} = m^{(i)}\, s^{(i)} \quad ; \quad \rho_o^{(i)}/\rho^{(i)} = \det(\mathbf{F}^{(i)}) = F^{(i)3} \tag{3b,c}$$

where $\rho_o^{(i)}$ is the density in the reference configuration, it follows that the system internal energy can be expressed as a function of the '2n' generalized coordinates $F^{(i)}$ and $S^{(i)}$

$$U = U(F^{(i)}, S^{(i)}) \tag{3d}$$

Hence the generalized conservative forces for the system may be calculated as

$$G^{(i)} = \frac{\partial U}{\partial F^{(i)}} = -3\, V_o^{(i)}\, P^{(i)}\, F^{(i)2} \quad ; \quad \theta^{(i)} = \frac{\partial U}{\partial S^{(i)}} \tag{3e,f}$$

where

$$\frac{P^{(i)}}{\rho^{(i)2}} = \frac{\partial u^{(i)}}{\partial \rho^{(i)}} \quad ; \quad \theta^{(i)} = \frac{\partial u^{(i)}}{\partial s^{(i)}} \tag{3g,h}$$

define the pressure ($P^{(i)}$) and temperature ($\theta^{(i)}$) for particle 'i'. Since equations (3a) through (3h) are exact, the generalized conservative forces may be calculated for any equation of state

$$P^{(i)} = P^{(i)}(\rho^{(i)}, s^{(i)}) \quad ; \quad \theta^{(i)} = \theta^{(i)}(\rho^{(i)}, s^{(i)}) \tag{3i,j}$$

derived from an appropriate internal energy function.

SHEAR AND BULK VISCOSITY

Particle methods normally incorporate a numerical viscosity, in order to avoid particle streaming effects and accurately reproduce shock waves (Monaghan and Gingold, 1983). In the present model there are two deformation modes which must be damped. For

the relative motion of particle centers of mass, a 'shear viscosity' force may be assumed to act on the mass centers. The simplest form for such a damping force is one which acts along a line joining the mass centers of two neighboring particles and is proportional to the relative velocities of the particles

$$\mathbf{f}^{(i)} = \sum_{j=1}^{n} \mu^{(i,j)} \{(\dot{\mathbf{c}}^{(i)} - \dot{\mathbf{c}}^{(j)}) \cdot (\mathbf{c}^{(i)} - \mathbf{c}^{(j)})\} (\mathbf{c}^{(i)} - \mathbf{c}^{(j)}) / (\dot{\mathbf{c}}^{(i)} - \dot{\mathbf{c}}^{(j)})^2 \qquad (4a)$$

where $\mu^{(i,j)}$ is the damping coefficient associated with the interaction of particles 'i' and 'j'. Previous work in shock simulations (Noh, 1978) suggests a damping coefficient of the form

$$\mu^{(i,j)} = c_o \, (1/2) \, (\rho^{(i)} \, c_s^{(i)} A^{(i)} + \rho^{(j)} \, c_s^{(j)} A^{(j)}) \, \Lambda[\zeta^{(i,j)}] \qquad (4b)$$

$$A^{(i)} = \pi \, h^{(i)2} \quad ; \quad c_s^{(i)} = \{ \, K^{(i)}/\rho^{(i)} \, \}^{1/2} \qquad (4c,d)$$

where $A^{(i)}$, $c_s^{(i)}$, and $K^{(i)}$ are the (variable) cross sectional area, sound speed, and bulk modulus of particle 'i', c_o is a dimensionless shear viscosity coefficient, and the step function 'Λ' is defined by

$$\Lambda[\zeta] = 1 \ \text{ for } \ \zeta \geq 0 \ ; \ \Lambda[\zeta] = 0 \ \text{ for } \ \zeta < 0 \qquad (4e)$$

$$\zeta^{(i,j)} = (h^{(i)}+h^{(j)}) - |\mathbf{c}^{(i)} - \mathbf{c}^{(j)}| \qquad (4f)$$

The step function ensures that only neighboring particles interact.

In order to damp the volumetric deformation mode of individual particles, a 'bulk viscosity' is required. Here a viscous 'pressure' (specifically an isotropic first Piola-Kirchoff stress) is introduced with the simple form

$$P^{B(i)} = - \nu^{(i)} \dot{F}^{(i)} \quad ; \quad \nu^{(i)} = c_1 \, \rho^{(i)} \, c_s^{(i)} \, h^{(i)} \qquad (5a,b)$$

where $\nu^{(i)}$ is a bulk viscosity with c_1 a dimensionless coefficient.

HEAT CONDUCTION

As in the case of the viscosity, accurate representation of shocks normally calls for the introduction of a heat conduction or artificial heat flux model (Noh, 1978). With the particle entropy serving as generalized coordinate, heat conduction is appropriately represented here using the entropy rate form

$$\dot{S}^{con(i)} = (1/\theta^{(i)}) \sum_{j=1}^{n} R^{(i,j)} (\theta^{(i)} - \theta^{(j)}) \tag{6a}$$

where $R^{(i,j)}$ is the overall heat transfer coefficient describing conduction between particles 'i' and 'j',

$$R^{(i,j)} = c_2 (1/2) (\rho^{(i)} c_s^{(i)} c_v^{(i)} A^{(i)} + \rho^{(j)} c_s^{(j)} c_v^{(j)} A^{(j)}) \Lambda[\zeta^{(i,j)}] \tag{6b}$$

with $c_v^{(i)}$ the (variable) specific heat for particle 'i' and c_2 a dimensionless heat conduction coefficient. Again only neighboring particles interact.

MECHANICAL AND THERMAL CONSTRAINTS

The adoption of a purely Lagrangian frame of reference has simplified the formulations in the preceding sections, but it somewhat complicates the treatment of constraints. In place of conventional energy balance equations, evolution equations for the entropy are introduced here, in the form of the nonholonomic constraints

$$\dot{S}^{(i)} = \dot{S}^{irr(i)} - \dot{S}^{con(i)} \tag{7a}$$

where $\dot{S}^{irr(i)}$ is the rate of irreversible entropy production in particle 'i'. The latter quantity is calculated from the energy dissipation rate ($\dot{W}^{(i)}$) associated with shear and bulk viscosity effects

$$\dot{S}^{irr(i)} = (1 / \theta^{(i)}) \dot{W}^{(i)} \tag{7b}$$

where

$$\dot{W}^{(i)} = \mathbf{f}^{(i)} \cdot \dot{\mathbf{c}}^{(i)} - V_o^{(i)} P^{B(i)} \dot{F}^{(i)} = \mathbf{f}^{(i)} \cdot \dot{\mathbf{c}}^{(i)} + G^{B(i)} \dot{F}^{(i)} \qquad (7c)$$

$$G^{B(i)} = c_1 \rho^{(i)} c_s^{(i)} h^{(i)} V_o^{(i)} \dot{F}^{(i)} \qquad (7d)$$

The coefficients of the generalized velocities ($\dot{\mathbf{c}}^{(i)}$ and $\dot{F}^{(i)}$) in the nonholonomic constraints (7a) will give rise to nonconservative generalized forces in the system level state equations.

The mechanical constraints on the particles arise from particle collisions. Specifically the generic particles 'i' and 'j' must satisfy the inequality constraints

$$|\mathbf{c}^{(i)} - \mathbf{c}^{(j)}| - \alpha (h^{(i)} + h^{(j)}) \geq 0 \quad ; \quad \alpha = (\pi/6)^{1/3} \qquad (8a,b)$$

where the factor α allows for 'body-centered' cubic packing of particles at the reference density. In rate form the last expression is

$$[(\mathbf{c}^{(i)} - \mathbf{c}^{(j)}) / |\mathbf{c}^{(i)} - \mathbf{c}^{(j)}|] \cdot (\dot{\mathbf{c}}^{(i)} - \dot{\mathbf{c}}^{(j)}) - \alpha \{ h_o^{(i)} \dot{F}^{(i)} + h_o^{(j)} \dot{F}^{(j)} \} \geq 0 \qquad (8c)$$

As is often the case, the inequality constraints introduced here call for a somewhat ad hoc treatment. In the next section they are accounted for by the introduction of Lagrange multipliers, which then give rise to additional generalized forces in the system level state equations.

HAMILTON'S EQUATIONS

The preceding description of stored energy functions and constraint equations allows for formulation of a system level Hamiltonian model (Ginsberg, 1988). The Hamiltonian (Π) for the system is

$$\Pi = T + U = \Pi (\mathbf{p}^{(i)}, \mathbf{c}^{(i)}, H^{(i)}, F^{(i)}, S^{(i)}) \qquad (9a)$$

The canonical form of Hamilton's equations is therefore (for i = 1, 2, ..., n)

$$\dot{\mathbf{p}}^{(i)} = - \frac{\partial \Pi}{\partial \mathbf{c}^{(i)}} + \mathbf{q}^{(i)} \quad ; \quad \dot{\mathbf{c}}^{(i)} = \frac{\partial \Pi}{\partial \mathbf{p}^{(i)}} \qquad (9b,c)$$

$$\dot{H}^{(i)} = -\frac{\partial \Pi}{\partial F^{(i)}} + Q^{F(i)} \quad ; \quad \dot{F}^{(i)} = \frac{\partial \Pi}{\partial H^{(i)}} \tag{9d,e}$$

$$0 = -\frac{\partial \Pi}{\partial S^{(i)}} + Q^{S(i)} \tag{9f}$$

where $\mathbf{q}^{(i)}$, $Q^{F(i)}$, and $Q^{S(i)}$ are the generalized nonconservative forces, which must be calculated here from the thermal and mechanical constraints. If $\gamma^{(i)}$ are Lagrange multipliers for the entropy evolution constraints (7a), and $\lambda^{(i,j)}$ are Lagrange multipliers for the particle interaction constraints (8c), it follows that

$$Q^{S(i)} = \gamma^{(i)} \tag{10a}$$

$$Q^{F(i)} = -(\gamma^{(i)}/\theta^{(i)})\, G^{B(i)} + G^{c(i)} \tag{10b}$$

$$\mathbf{q}^{(i)} = -(\gamma^{(i)}/\theta^{(i)})\, \mathbf{f}^{(i)} + \mathbf{f}^{c(i)} \tag{10c}$$

where the contributions

$$\mathbf{f}^{c(i)} = \sum_{j=1}^{n} \lambda^{(i,j)} (\mathbf{c}^{(i)} - \mathbf{c}^{(j)}) / |\mathbf{c}^{(i)} - \mathbf{c}^{(j)}| \tag{11a}$$

$$G^{c(i)} = -\sum_{j=1}^{n} \alpha\, h_o^{(i)}\, \lambda^{(i,j)} \tag{11b}$$

arise from the mechanical constraints and the remaining terms on the right hand sides of equations (10) are due to the nonholonomic entropy evolution constraints.

For an efficient solution procedure, elimination of the Lagrange multipliers is of course desirable. Note that the last of Hamilton's equations requires

$$\theta^{(i)} = Q^{S(i)} \tag{11c}$$

and hence identifies the temperature as the Lagrange multiplier $\gamma^{(i)}$ for the 'ith' entropy evolution constraint equation. To avoid the considerable complication of taking the $\lambda^{(i,j)}$ to be unknowns in a differential-algebraic formulation of the system level model, it is advantageous to represent the latter quantities as penalty forces, by assuming

$$\lambda^{(i,j)} = k^{(i,j)} [\, \alpha\, (h^{(i)} + h^{(j)}) - |\mathbf{c}^{(i)} - \mathbf{c}^{(j)}| \,]\, \Lambda[\zeta^{c(i,j)}] \tag{11d}$$

where the penalty stiffness is (Hallquist, 1983)

$$k^{(i,j)} = c_3 \max\{ K^{(i)} A^{(i)2} / V^{(i)}, K^{(j)} A^{(j)2} / V^{(j)} \} \tag{11e}$$

and

$$\zeta^{c(i,j)} = \alpha (h^{(i)}+h^{(j)}) - |\mathbf{c}^{(i)} - \mathbf{c}^{(j)}| \tag{11f}$$

with c_3 a dimensionless penalty stiffness. Note that the use of a Lagrange multiplier technique here serves the essential task of properly scaling the contributions of the collision forces $\lambda^{(i,j)}$ in the evolution equations for both the center of mass momenta and the distributed momenta.

Combining the expressions (9) through (11) yields the final form of Hamilton's equations as

$$\dot{\mathbf{p}}^{(i)} = - \mathbf{f}^{(i)} + \mathbf{f}^{c(i)} \tag{12a}$$

$$\dot{\mathbf{c}}^{(i)} = m^{(i)-1} \mathbf{p}^{(i)} \tag{12b}$$

$$\dot{H}^{(i)} = - G^{(i)} - G^{B(i)} + G^{c(i)} \tag{12c}$$

$$\dot{F}^{(i)} = J^{(i)-1} H^{(i)} \tag{12c}$$

$$\dot{S}^{(i)} = \dot{S}^{irr(i)} - \dot{S}^{con(i)} \tag{12d}$$

Perusal of the preceding sections shows that the relations (12) are in fact explicit rate equations for the state variables $\mathbf{p}^{(i)}$, $\mathbf{c}^{(i)}$, $H^{(i)}$, $F^{(i)}$, and $S^{(i)}$ (i = 1, 2, ..., n).

NUMERICAL IMPLEMENTATION

Several other points relevant to numerical implementation should be noted. Symmetry boundary conditions may be applied by extending the penalty approach used in the preceding sections to account for particle collisions. For example, for model symmetry about the plane z = 0, it is appropriate to introduce additional constraint forces for each particle in the form

$$\mathbf{f}^{z(i)} = c_3 \, (K^{(i)} \, A^{(i)2} / \, V^{(i)}) \, [\, \alpha \, h^{(i)} - \min\{ \, \alpha \, h^{(i)}, \, \mathbf{c}^{(i)} \cdot \, \mathbf{i}_z \, \}] \, \mathbf{i}_z \qquad (13a)$$

$$G^{z(i)} = - \, c_3 \, \alpha \, h_o^{(i)} \, (K^{(i)} \, A^{(i)2} / \, V^{(i)}) \, [\, \alpha \, h^{(i)} - \min\{ \, \alpha \, h^{(i)}, \, \mathbf{c}^{(i)} \cdot \, \mathbf{i}_z \, \}] \qquad (13b)$$

where $\mathbf{i}_z$ is a fixed basis vector.

If a Runge-Kutta or leap-frog (Libersky et. al, 1993) algorithm is used to integrate the system state equations, a variable time step may selected as follows:

$$\Delta t = (1/c_4) \, w_{max}^{-1/2} \qquad (14a)$$

$$w_{max} = \max\{ \, w_k^{(i)} \text{ for } k = 1, 2, 3, ..., 6 \text{ and } i = 1, 2, 3, ..., n \, \} \qquad (14b)$$

where c_4 is a dimensionless constant and the $w_k^{(i)}$ are squared frequencies calculated using

$$w_k^{(i)} = |\dot{p}_k^{net}| / (m^{(i)} h_{min}) \quad ; \quad k = 1, 2, 3 \qquad (14c)$$

$$w_4^{(i)} = \mathbf{p}^{(i)2} / (m^{(i)} h_{min})^2 + K^{(i)} / (\rho^{(i)} h_{min}^2) \qquad (14d)$$

$$w_5^{(i)} = |\dot{H}^{net}| / J^{(i)} \quad ; \quad w_6^{(i)} = |\dot{S}^{net}| / (m^{(i)} c_v^{(i)})^2 \qquad (14e,f)$$

$$h_{min} = \min\{ \, h^{(i)} \text{ for } i = 1, 2, 3, ..., n \, \} \qquad (14g)$$

The net rates of change indicated in the last set of equations are defined as the right hand sides of equations (12). The first four time step limits of (14c) and (14d) are used by Monaghan and co-workers (Monaghan, 1992 and Monaghan and Lattanzio, 1985) while the last two are appended here for the distributed momentum and entropy evolution equations.

Finally linked lists (Hockney and Eastwood, 1981) may be used to identify the nearest neighbors of each particle and thereby greatly reduce the effective lengths of the summations in equations (4a), (6a), (11a), and (11b). Since each particle has a time varying radius, the cell size used in the construction of linked lists is determined by the maximum particle radius

$$h_{max} = \max\{ h^{(i)} \text{ for } i = 1, 2, 3, ..., n \} \qquad (14h)$$

EXAMPLE SIMULATIONS

The example simulations discussed here were run using a three dimensional, vectorized and autotasked implementation of the model discussed in the preceding sections of the report. All of the examples take

$$c_0 = 0.05 \; ; \; c_1 = 1.00 \; ; \; c_2 = 1.00 \; ; \; c_3 = 25.0 \qquad (15)$$

The first example is a one dimensional wall shock problem (Noh, 1978), applied to an ideal gas with density shift, described by the equation of state

$$u^{(i)} = c_v (\theta^{(i)} - \theta_0) \quad ; \quad P^{(i)} = (Z-1)\, c_v\, \theta^{(i)} (\rho^{(i)} - \rho_0) \qquad (16a,b)$$

$$\theta^{(i)} = \theta_0 \exp[(s^{(i)} - s_0)/c_v] \, [\rho^{(i)}/\rho_0]^{(Z-1)} \exp[(Z-1)(\rho_0/\rho^{(i)}-1)] \qquad (16c)$$

where Z is the ratio of specific heats, θ_0 is the reference temperature, and s_0 is the reference entropy. Parameters of the simulation are shown in Table 1. This problem represents the collision (at position x = 0 and time t = 0) and shock compression of two particle streams with initial conditions

$$\rho = 1.0 \quad ; \quad s = 1.0 \quad ; \quad H = 0.0 \quad ; \quad F = 1.0 \qquad (17a)$$

$$\dot{c} = +1.0 \text{ for } x < 0 \text{ and } \dot{c} = -1.0 \text{ for } x > 0 \qquad (17b)$$

Post shock conditions calculated from the Rankine-Hugoniot relations (Cole, 1948) are

$$v = 0.0 \quad ; \quad \rho = 2.618 \quad ; \quad P = 1.618 \quad ; \quad \theta = 1.5 \qquad (18)$$

Figures 1 through 4 show good agreement of the exact and numerical velocity, density, pressure, and temperature distributions at t = 0.8, with minimal shock smearing.

The second example represents the oblique (three dimensional) impact of a spherical projectile on a Whipple shield at approximately seven kilometers per second. Both the projectile and the shield were taken to be aluminum, with the material described by a Mie-Grüneisen equation of state. Parameters of the simulation are shown in Table 2. Figure 5 shows the simulation results. Simulation of the shield perforation and debris cloud evolution to initial impact on the wall plate required about one CPU hour on a Cray T916.

The third example represents the oblique impact of a spherical projectile on a multi-plate shield at approximately seven kilometers per second. Again both the projectile and shield materials were taken to be aluminum, and a Mie-Grüneisen equation of state was used. Parameters of the simulation are shown in Table 3. Figure 6 shows impact of the projectile and evolution of the debris cloud as successive shields are perforated. The simulation required about six CPU hours on a Cray T916, without rezoning, in order to reach initial impact on the wall plate.

CONCLUSION

The present report has presented a new numerical method for hydrodynamics simulation, developed using a fully discrete Hamiltonian description of a system of thermomechanically interacting physical particles. It relies exclusively on energy concepts, as opposed to Eulerian or Lagrangian interpolation functions and partial differential equation descriptions of the continuum dynamics. The kinematics, stored energy functions, and constraints are all formulated in a fully Lagrangian frame of reference. In addition it makes uniform use of entropy variables in accounting for all conservative and non-conservative thermal effects. The preceding combination of modeling features makes the present work a distinct departure from existing PIC, SPH, and EFG methods, as well as more conventional Lagrangian, Eulerian, and arbitrary Lagrangian-Eulerian hydrocodes. Specific advantages of the method include: (1) the simplicity of the formulation, (2) the

avoidance of stability problems which arise with the essentially Eulerian potential energy formulations of generalized SPH, and (3) the elimination of the particle-to-grid and grid-to-particle mappings associated with generalized PIC.

The present work does not address effects such as multi-phase flow (Monaghan and Kocharyan, 1995) and fragmentation (Benz and Asphaug, 1995) which have been incorporated into alternative modeling schemes, as those methods evolved. However the fully Lagrangian formulation used here is clearly amenable to the treatment of multi-phase problems and material nonlinearities in a hydrodynamic context.

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Table 1. Wall shock simulation

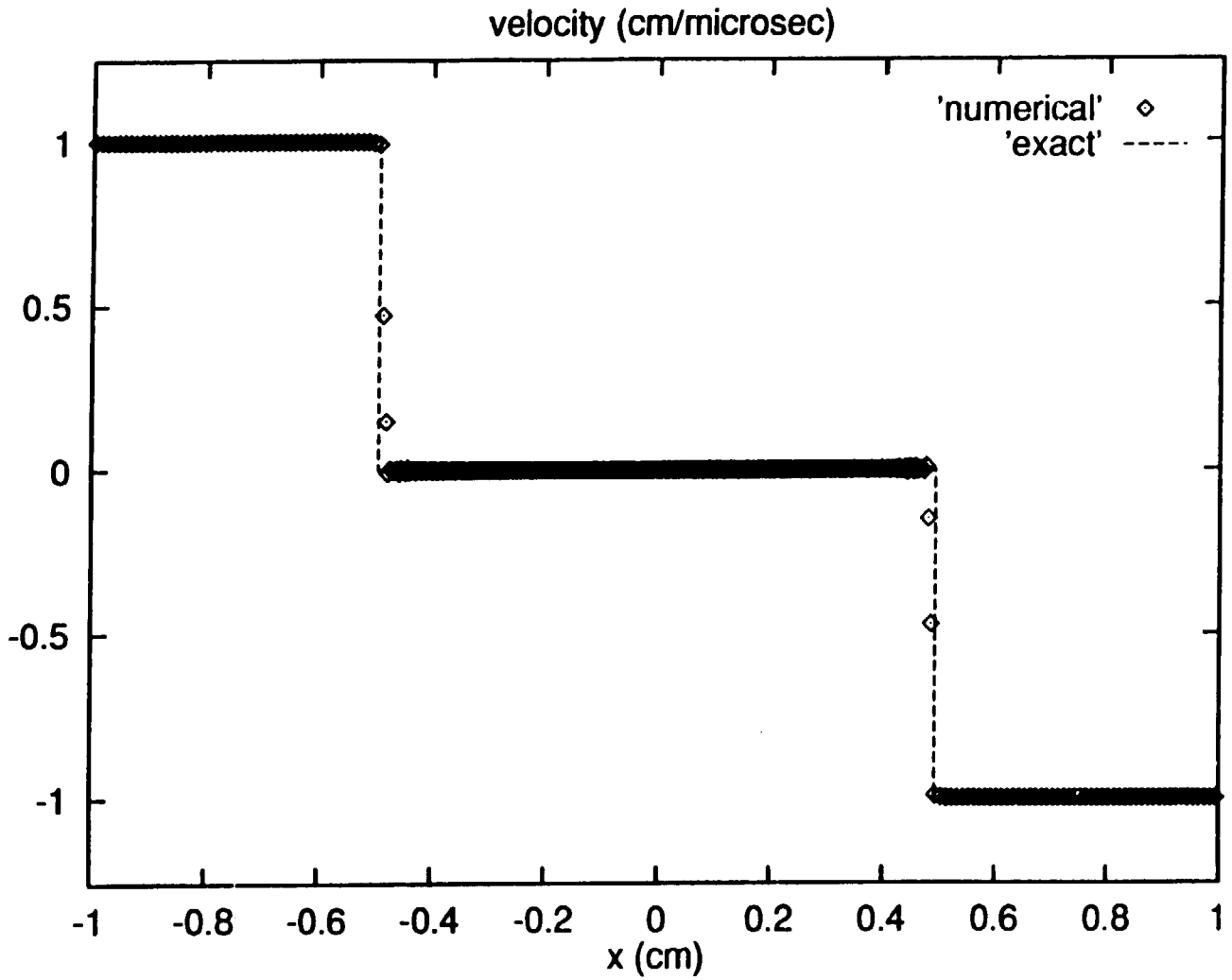
Reference density	=	1.0
Reference temperature	=	1.0
Reference entropy	=	1.0
Specific heat	=	1.0
Ratio of specific heats	=	5/3
Number of particles ($-2 \leq x \leq +2$)	=	400

Table 2. Whipple shield impact simulation

Projectile diameter	=	0.500 cm
Shield thickness	=	0.127 cm
Impact velocity	=	0.695 cm/μsec
Impact obliquity	=	30 degrees
Material	=	aluminum
Equation of state type	=	Mie-Grüneisen
Reference density	=	2.70 g/cm^3
Reference temperature	=	293°K
Specific heat	=	0.884 x 10^{-5} Mb-cm^3/(g-°K)
Initial Grüneisen gamma	=	1.97
Hugoniot slope coefficient	=	1.339
Bulk sound speed	=	0.5386 cm/μsec
Number of particles	=	3,136
Total simulation time	=	17.0 μsec
CPU time (Cray T916)	=	1.03 hrs
Number of time steps	=	10,000

Table 3. Multi-plate shield impact simulation

Projectile diameter	=	0.320 cm
Plate thickness	=	0.010 cm
Plate spacing	=	2.50 cm
Number of plates	=	4
Impact velocity	=	0.695 cm/μsec
Impact obliquity	=	30 degrees
Material	=	aluminum
Equation of state type	=	Mie-Grüneisen
Reference density	=	2.70 g/cm^3
Reference temperature	=	293°K
Specific heat	=	0.884 x 10^{-5} Mb-cm^3/(g-°K)
Initial Grüneisen gamma	=	1.97
Hugoniot slope coefficient	=	1.339
Bulk sound speed	=	0.5386 cm/μsec
Maximum number of particles	=	23,468
Total simulation time	=	18.0 μsec
Number of time steps	=	18,323
CPU time (Cray T916)	=	6.01 hrs


velocity (cm/microsec)
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'exact'
1
0.5
0
-0.5
-1
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
x (cm)

Figure 1

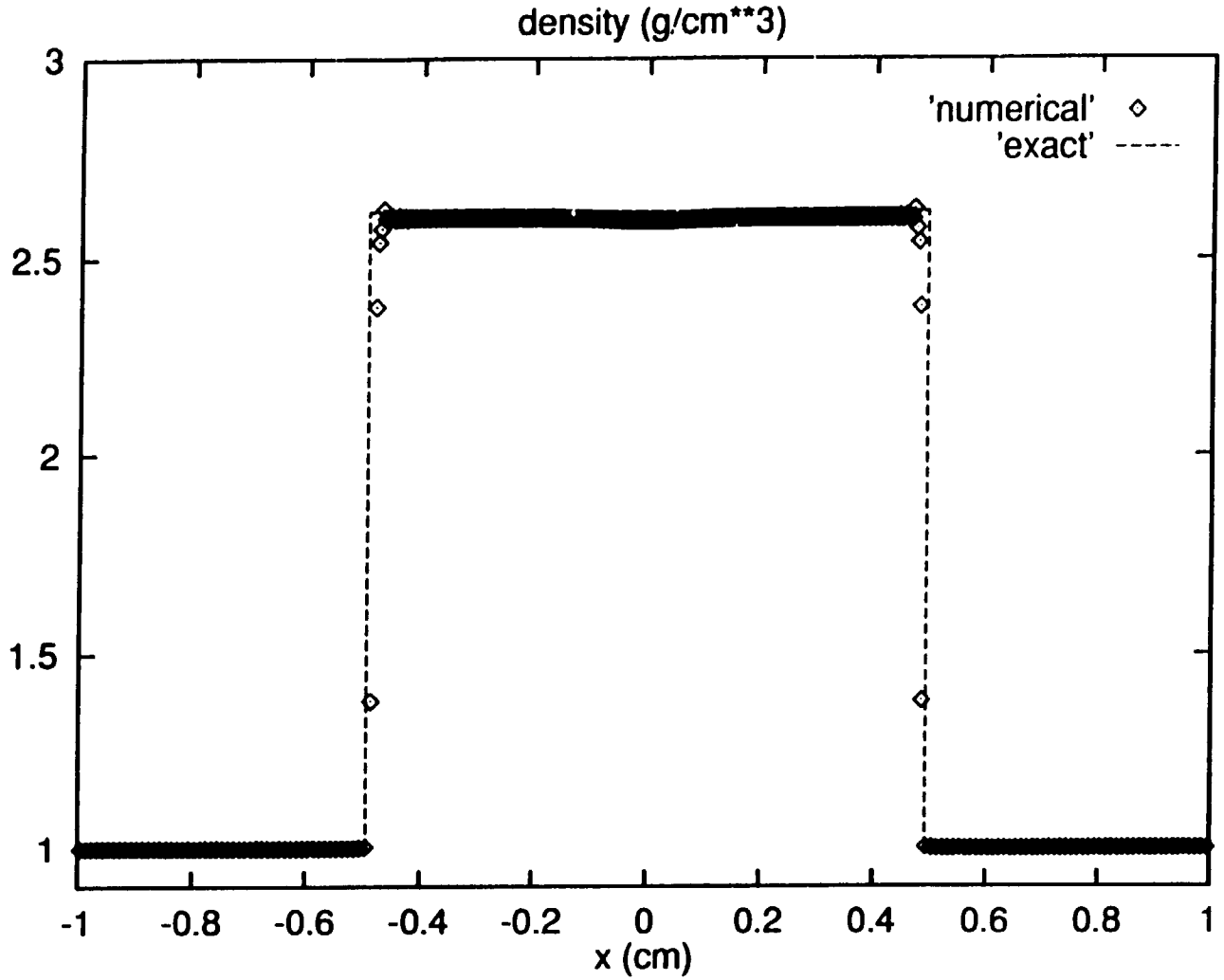

density (g/cm**3)
'numerical'
'exact'
3
2.5
2
1.5
1
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
x (cm)

Figure 2

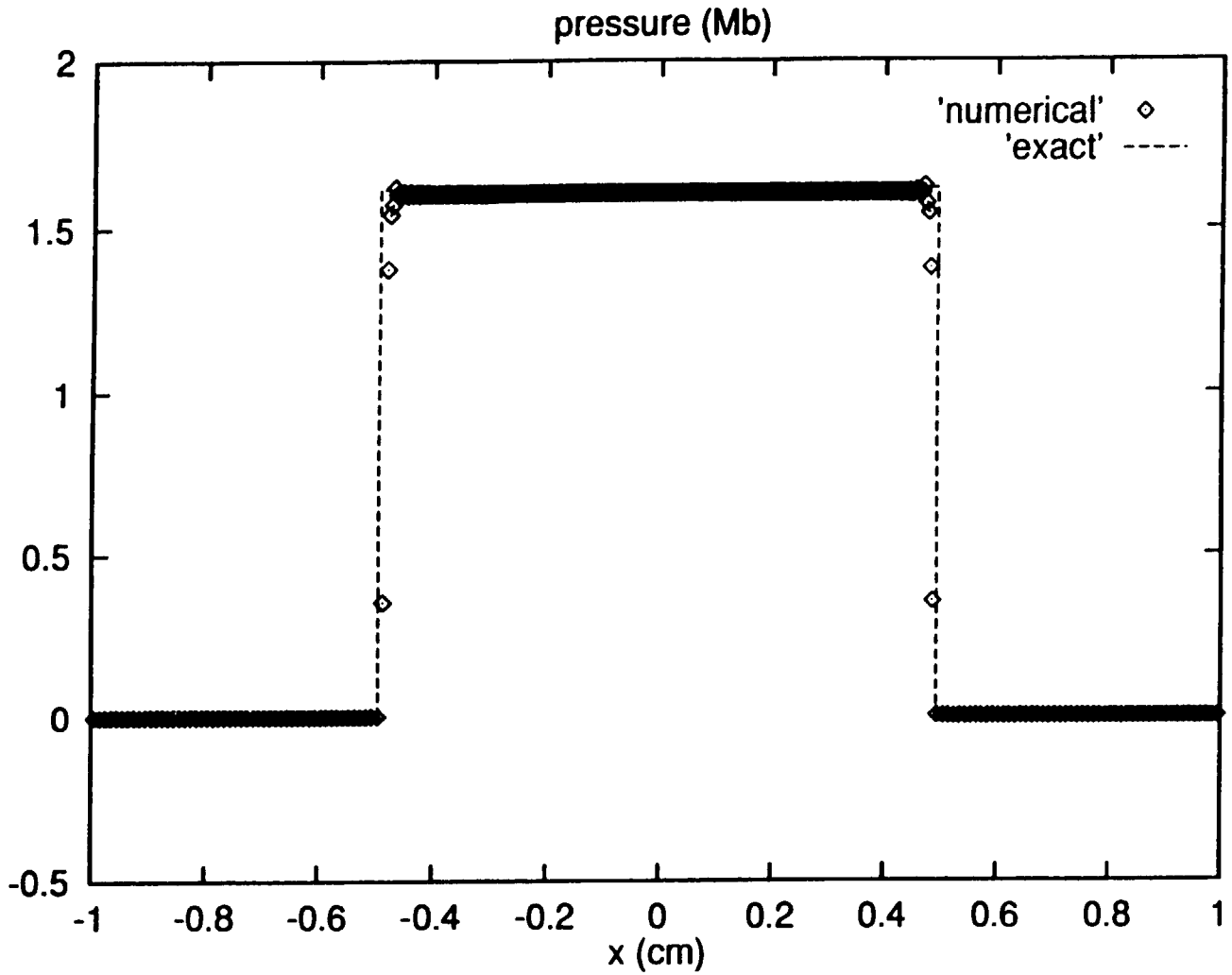

pressure (Mb)
'numerical'
'exact'
2
1.5
1
0.5
0
-0.5
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
x (cm)

Figure 3

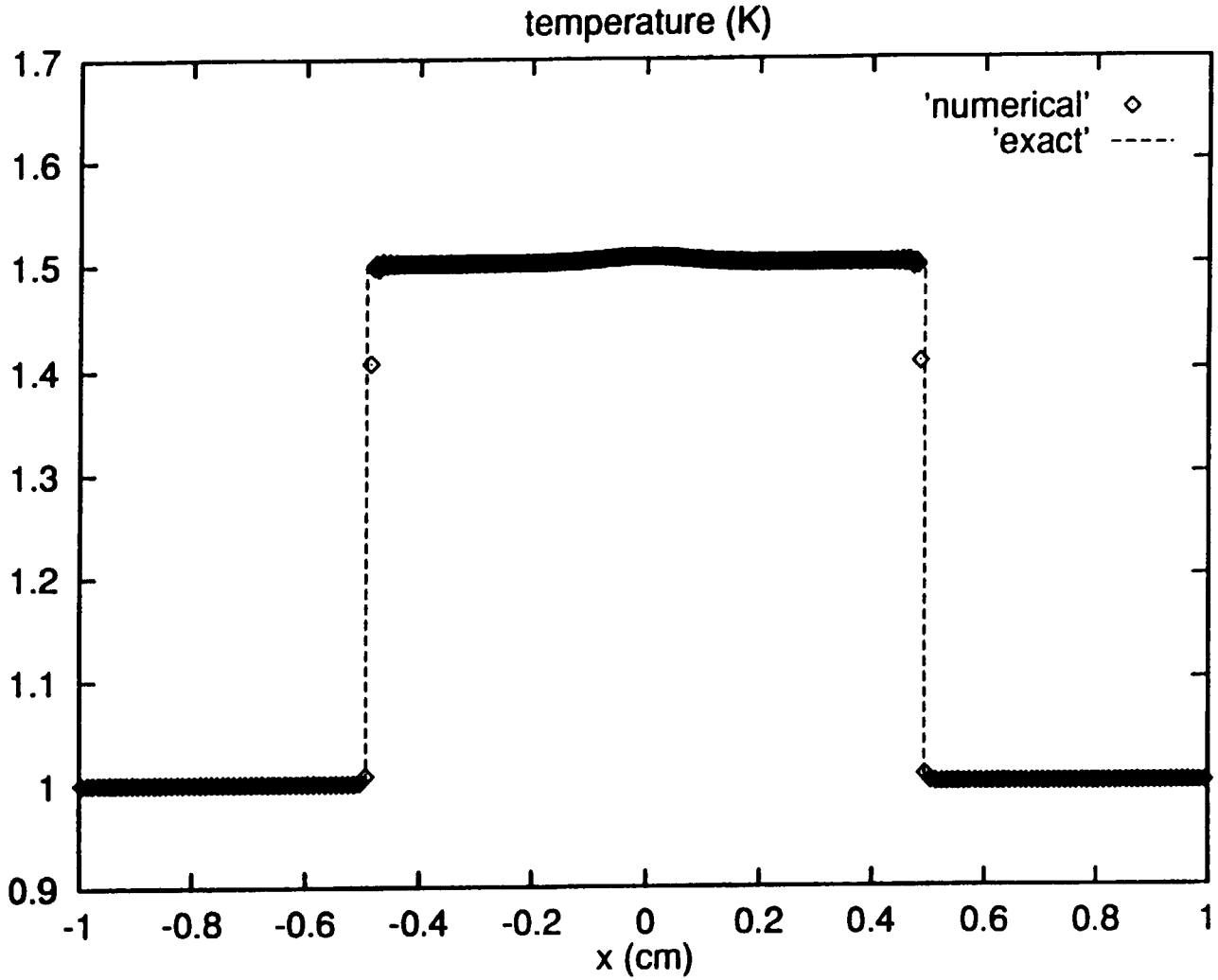
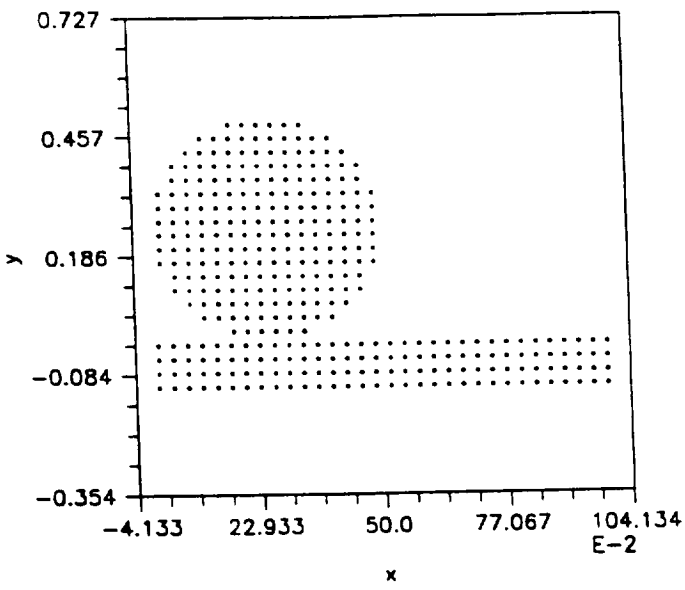
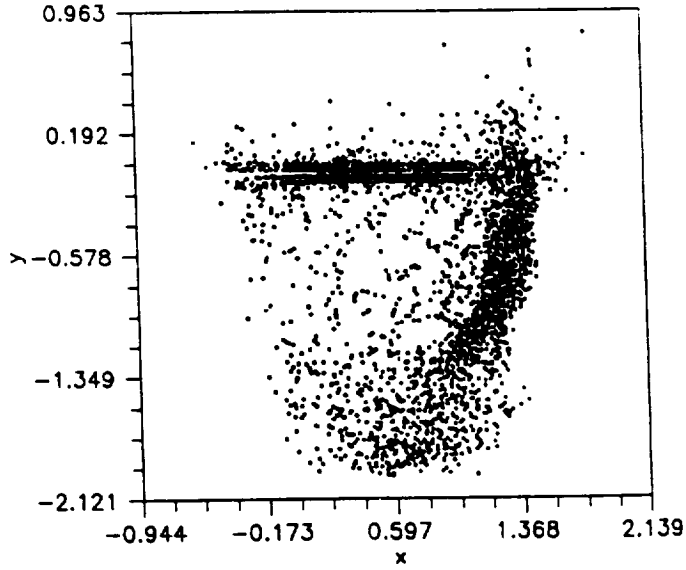
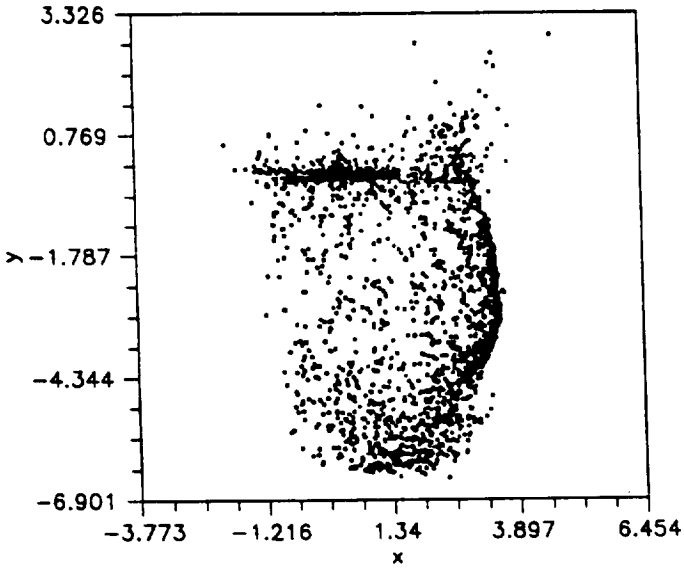

temperature (K)
'numerical'
'exact'
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0.9
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
x (cm)

Figure 4


0.727
0.457
y 0.186
−0.084
−0.354
−4.133
22.933
50.0
77.067
104.134
E−2
x


0.963
0.192
y −0.578
−1.349
−2.121
−0.944
−0.173
0.597
1.368
2.139
x


3.326
0.769
y −1.787
−4.344
−6.901
−3.773
−1.216
1.34
3.897
6.454
x

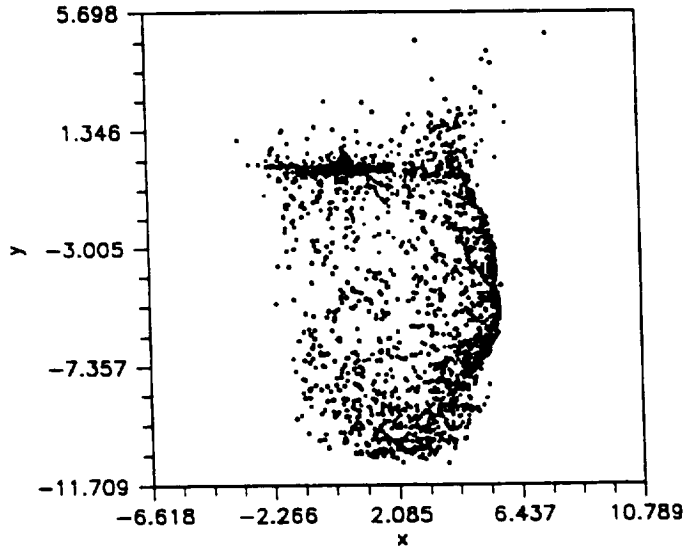
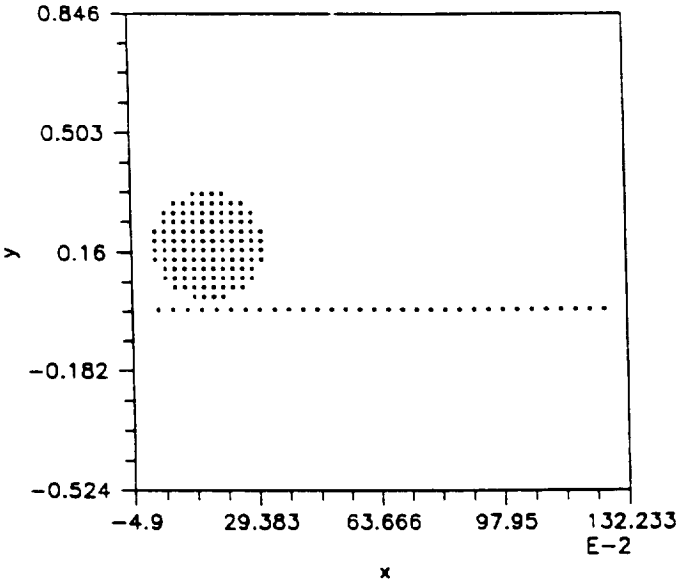
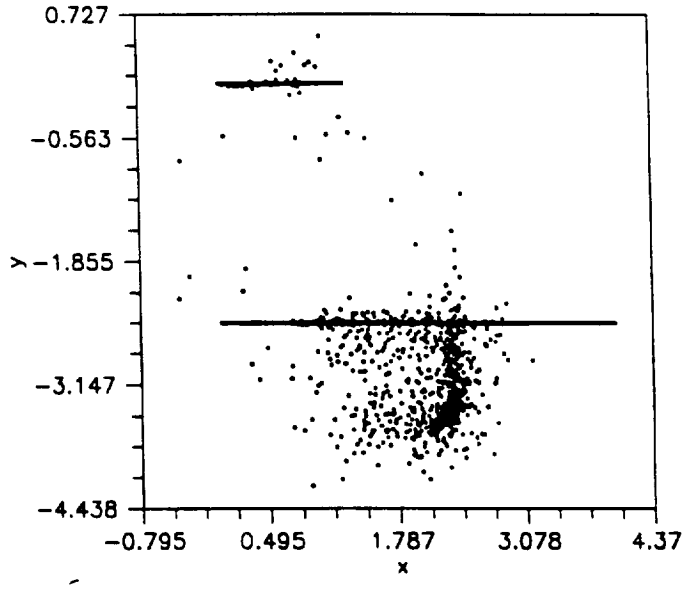
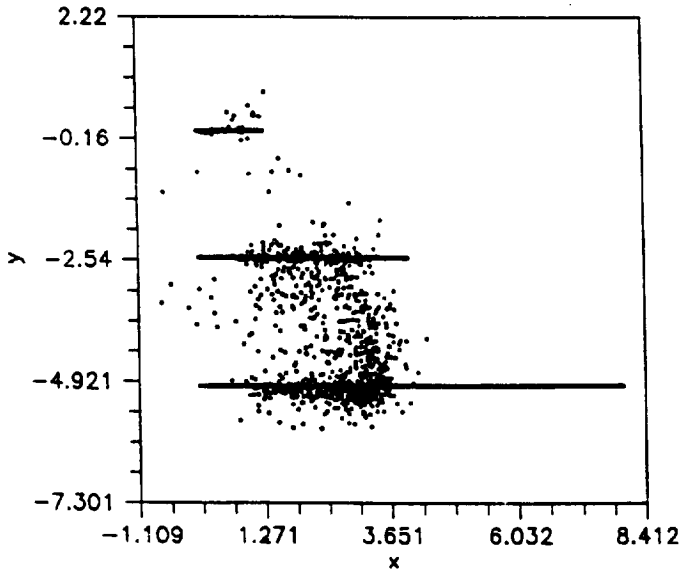

5.698
1.346
y −3.005
−7.357
−11.709
−6.618
−2.266
2.085
6.437
10.789
x

Figure 5


0.846
0.503
0.16
-0.182
-0.524
y
-4.9
29.383
63.666
97.95
132.233
E-2
x


0.727
-0.563
-1.855
-3.147
-4.438
y
-0.795
0.495
1.787
3.078
4.37
x


2.22
-0.16
-2.54
-4.921
-7.301
y
-1.109
1.271
3.651
6.032
8.412
x

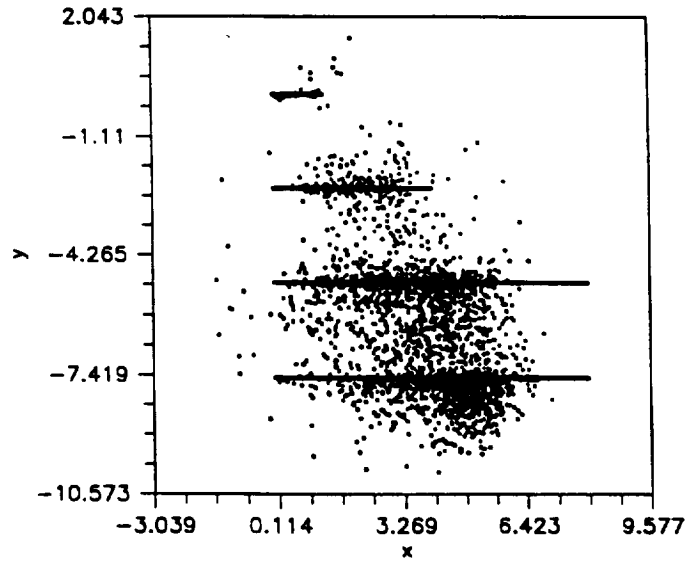

2.043
-1.11
-4.265
-7.419
-10.573
y
-3.039
0.114
3.269
6.423
9.577
x

Figure 6